THE
MATHEMATICAL GAZETTE.

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ON THE REPRESENTATION OF IMAGINARY POINTS BY REAL POINTS ON A PLANE.

(*Continued from the* Mathematical Gazette *for May*, 1903.)

I wish to express my great indebtedness to Mr. P. J. Heawood, of Durham University, for his careful and suggestive criticism of the contents of this paper, without, however, making him responsible for any statement in it with which he may disagree.—A. LODGE.

FOR the sake of those readers who have not seen the first part of this paper, and to whom the whole subject is new, I may state that the whole argument is based on the fact that any imaginary point such as $(6+i,\ 4+3i)$ has a simple vector $x+iy=(6+i)+i(4+3i)=3+5i$, and therefore may be vectorially plotted on to, or represented by, the real point [in this case (3, 5)] which has the same vector. This real point is thus the *representative* of all the imaginary points which have the same vector. The paper deals with the consequent relations between the representatives of conjugate imaginaries, especially when the imaginaries satisfy the equation of some given curve: in this case the two real points representing such a pair of conjugate imaginaries are called images of each other in the curve.

Every point P in the plane has n such images in a curve of the n^{th} degree. If two images of P coincide, P is a focus—unless the two images coincide with P itself, in which case it is a double point on the curve.

THERE are a number of interesting theorems relating to the pair of representatives of a pair of conjugate imaginary points.

The first thing to do is to establish certain fundamental equations by means of which when any two of the four points are given the others can be found.

Let P, Q be the real points, with coordinates (α, β) and (γ, δ) respectively; and let I, J be the corresponding imaginary points with coordinates (x_1, y_1) and (x_2, y_2) respectively.

Then, since by supposition an imaginary point and its representative have the same *vectors*, we have the two equations

$$x_1+iy_1=\alpha+i\beta, \quad\text{.............................(1)}$$

and
$$x_2+iy_2=\gamma+i\delta. \quad\text{.............................(2)}$$

Two other important equations can be obtained by considering the values of what may be called the anti-vectors of the points, *i.e.* the values of $x-iy$.

It is obvious that x_1+iy_1 and x_2-iy_2 differ throughout in the sign of i, and in that only; for if the coordinates of I are of the form $m+ip$, $n+iq$, those of J will be of the form $m-ip$, $n-iq$.

Hence, $$x_2-iy_2=\alpha-i\beta, \qquad (3)$$

and, similarly, $$x_1-iy_1=\gamma-i\delta. \qquad (4)$$

These four equations are sufficient to find the other two points when two of them are given.

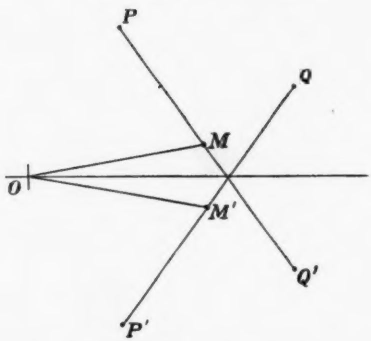
They can be illustrated graphically as shown below; for though the imaginary points themselves cannot be shown (unless they are considered as coinciding with the representative points), their *coordinates* can be so represented, since x and iy are merely vectors whose addition leads us to the point (x, y).

Let P be the point (α, β),

P' " " $(\alpha, -\beta)$,

Q " " (γ, δ),

Q' " " $(\gamma, -\delta)$.



Then, if M is the mid-point of PQ', we have, by addition of vectors,

$$OM+MP=OP,$$
$$OM-MP=OQ'.$$

These are the equations:

$$x_1+iy_1=\alpha+i\beta, \qquad (1)$$
$$x_1-iy_1=\gamma-i\delta. \qquad (4)$$

Hence x_1 is the vector OM,

and iy_1 " " MP.

Similarly, if M' is the mid-point of QP', we see, from equations (2) and (3), that x_2 is the vector OM',

and iy_2 " " $M'Q$.

[In the diagram, P is at (3, 5), and Q is at (9, 3); and the coordinates of I are $(6+i, 4+3i)$; while those of J are $(6-i, 4-3i)$. So that the vector of

$$OI=6+i+i(4+3i)=3+5i=OP,$$

and the vector of $OJ=6-i+i(4-3i)=9+3i=OQ$.]

It follows, from the above, that any pair of points P, Q in the plane can represent some one pair of conjugate imaginary points, and also that to any point P correspond an infinite number of imaginary points given by the equation (1), the conjugates of such points having representative points Q, which cover the whole plane.

Further, there will be one imaginary point given by equation (1) whose coordinates will be infinite, and the conjugate of that point will be situated at infinity. These points, which may be called Ω and Ω', are the circular points at infinity. All the points given by equation (1), including Ω, appear to be piled on P, and Ω itself having infinite coordinates may be considered as being the meeting point of all the lines $x+iy=$constant.

If any pair of conjugate imaginary points I, J are points of a curve $f(x, y)=0$, their representative points, P, Q, will, as a rule, not be situated on the curve, but they will have in many cases interesting relations with the curve. I propose to call them images of each other in the curve. In a curve of the n^{th} degree we can show that there will be n imaginary points of the curve, $I_1, I_2, \ldots I_n$, corresponding to any given representative point P which is not itself a point on the curve, and these points will have n conjugate points $J_1, \ldots J_n$, whose representative points $Q_1, \ldots Q_n$ will be scattered over the plane, *i.e.* every point P will have n images in a curve of the n^{th} degree.

For, if $\alpha+i\beta$ be the vector common to any given point P and to all the I points represented by P, then if we denote the vector of one of the conjugate J points by z and its anti-vector by ω, we shall have, for any of these J points

$$x+iy=z,$$
$$x-iy=\alpha-i\beta=\omega\,;$$

ω being given, and z being required.

From these we obtain

$$2x=z+\omega,$$
$$2iy=z-\omega\,;$$
$$\therefore\ 2y=i(\omega-z).$$

Substituting these values in $f(x, y)=0$, we obtain the required equation in z, viz.:

$$f\left\{\frac{\omega+z}{2},\quad \frac{i(\omega-z)}{2}\right\}=0,$$

the solution of which gives the vectors of the images of P, since J and Q have the same vector.

In this equation, the highest power of z involved is z^n, consequently the equation will be of the n^{th} degree unless the highest terms cancel. In such exceptional case, one or more of the values of z must be infinite, and consequently the points Ω and Ω' will be imaginary points of the curve.

Including the infinitely distant points, we may therefore say, generally, that to each point P correspond n images $Q_1, \ldots Q_n$. These points represent n imaginary points $J_1, \ldots J_n$ of the curve, the conjugates of which are $I_1, \ldots I_n$, all of which have the same vector as P.

[The above equation between z and ω may be called the z, ω equation of the curve, z and ω being respectively the vector and anti-vector of any point of the curve. It is important to notice, however, in the case of an imaginary point, that whereas z is also the vector of the real representative of the imaginary point, ω is the anti-vector of the real representative of the *conjugate* imaginary point.]

Various interesting special cases suggest themselves. Thus, if P is on the curve, one of the images will coincide with it; if it is a double point on the curve, two of its images will coincide with it, and so on. It is possible, however, for two of the images to coincide with each other without

coinciding with P, whether P is on the curve or not; in such cases P will be a focus of the curve, for the definition of a focus is that two of its I points are coincident,* which necessitates the corresponding Q points being also coincident. If one or more images of every point be at infinity, the point, if any, which has *another* infinitely distant image will be a focus.†

Again, considering special curves, I showed in my first paper that if $f(x, y)=0$ is a straight line, any pair of images, P and Q, are perpendicularly equidistant from the line on opposite sides of it; *i.e.* they are, as it were, 'optical' images in the line, the distance of each from the line being equal to the modulus of the purely imaginary part of the coordinates of the corresponding imaginary points I and J. This line may be called the axis of the pair of conjugate points I and J. Hence every imaginary point I has a representative point P and an axis, the axis being the perpendicular line midway between P and its image Q (Q being the representative of the conjugate imaginary point J).

If $f(x, y)=0$ is a circle, we can show, as follows, that P and Q are inverse points with regard to the circle. For, taking the equation $x^2+y^2=r^2$, we find that its z, ω equation is $z\omega=r^2$.

Hence, if P is at (α, β), and ω is its anti-vector $\alpha-i\beta$, z will be the vector of Q, and is given by the equation

$$z=\frac{r^2}{\omega}=\frac{r^2}{\alpha-i\beta}=\frac{r^2}{\alpha^2+\beta^2}(\alpha+i\beta).$$

Therefore Q is at $\left(\frac{r^2\alpha}{\alpha^2+\beta^2}, \frac{r^2\beta}{\alpha^2+\beta^2}\right)$, which is obviously the inverse of (α, β).

It is important to notice that although a circle is a curve of the second degree, each point in the plane has only one finite image, the equation in z being of the first degree. The other image is at an infinite distance. The reason is that though x^2 and y^2 both contain z^2, x^2+y^2 does not, being equal to $z\omega$, the z^2 terms cancelling out. Thus Ω and Ω' are imaginary points of the circle.

In the same way, any curve for which every point in the plane has an infinitely distant image must have x^2+y^2 as a factor of the terms of the n^{th} degree, where n is the degree of the curve, and every point will have two infinitely distant images if $(x^2+y^2)^2$ is a factor of the n^{th} degree terms, and if either the terms of the $(n-1)^{\text{th}}$ degree are missing, or x^2+y^2 is a factor of them. The first set of curves are called circular, since Ω and Ω' are imaginary points of them; and the second set are called bicircular, Ω and Ω' being imaginary double points of them. Similarly, we may have tricircular curves, and so on. In any of these cases, if any special point has *another* infinitely distant image, that point will be a focus, as already stated.

The conditions for foci may be illustrated by finding the vector equation of the images of any point (α, β) in the ellipse $b^2x^2+a^2y^2=a^2b^2$, and showing that they will be coincident if (α, β) is a focus, and in no other case.

The z, ω equation of the ellipse is

$$b^2(z+\omega)^2-a^2(z-\omega)^2=4a^2b^2,$$

which reduces to
$$z^2-2\frac{a^2+b^2}{a^2-b^2}z\omega+\omega^2+\frac{4a^2b^2}{a^2-b^2}=0,$$

whence
$$z=\frac{a^2+b^2}{a^2-b^2}\omega\pm\frac{2ab}{a^2-b^2}\sqrt{(\omega^2-a^2e^2)}.$$

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* In other words, (α, β) is a focus if $x+iy=\alpha+i\beta$ is a tangent, which necessitates $x-iy=\alpha-i\beta$ being also a tangent.

† In this case $x+iy=\alpha+i\beta$ is an asymptote in the direction of Ω, and $x-iy=\alpha-i\beta$ is an asymptote in the direction of Ω'.

This equation gives the required images, and shows that they will be coincident if $\omega = \pm ae$; *i.e.* if $\alpha = \pm ae$, $\beta = 0$, and in no other case: which proves the proposition.

In finding the points of intersection of two curves, if any of the points are imaginary they will occur in pairs of conjugate points, whose real representatives will be images in each of the curves, as above defined. If we work with vectors, these real points will be given by the same equation as if they were points on the curves.

The method will be to eliminate x and y between the equation $z = x + iy$ and the equations of the two curves. The resulting equation in z will give the vectors of all the points of intersection, real and imaginary.

We will take as an important illustration of this method the following problem:

To find the pair of images common to an ellipse and one of its directrices, and to show that the director circle of the ellipse has the same two points as images.

Taking the three equations $\quad z = x + iy,$

$$b^2x^2 + a^2y^2 = a^2b^2,$$

$$ex = a,$$

we easily find $\quad z = \dfrac{a}{e} \mp \dfrac{b^2}{ae} = ae, \text{ or } \dfrac{a^2+b^2}{ae};$

$$\therefore \text{ the required pair of images are } (ae, 0) \text{ and } \left(\frac{a^2+b^2}{ae}, 0\right).$$

It is obvious that the equation $x^2 + y^2 = a^2 + b^2$; *i.e.* $z\omega = a^2 + b^2$, is satisfied by the vector and anti-vector of each of the imaginary points represented by these same points.

This problem has been introduced because it connects the foci of the ellipse with its director circle.

We have seen that the foci of a curve are points which have two coincident images in the curve. They are distinguishable from double points on the curve by the fact that the double images are not coincident with the points themselves. We will now find the point or points which have two coincident images in any curve $f(x, y) = 0$.

In this case it will be more convenient to denote the vector of the point itself by z, and to use ω to denote the anti-vector of its image.

The z, ω equation of the curve is

$$f\left\{\frac{\omega+z}{2}, \frac{i(\omega-z)}{2}\right\} = 0.$$

Denoting this, for brevity, as $u = 0$, we must have, as the condition for two equal values of ω, the two simultaneous equations,

$$u = 0, \ \frac{du}{d\omega} = 0.$$

Now $\quad \dfrac{du}{d\omega} = \dfrac{du}{dx} \cdot \dfrac{dx}{d\omega} + \dfrac{du}{dy} \cdot \dfrac{dy}{d\omega}$, z being constant,

where $\quad 2x = z + \omega$

and $\quad 2iy = z - \omega$;

$$\therefore \ \frac{du}{d\omega} = \frac{1}{2}\left(\frac{du}{dx} + i\frac{du}{dy}\right).$$

Hence we may write the two simultaneous equations as

$$\left.\begin{aligned} u&=0,\\ \frac{du}{dx}+i\frac{du}{dy}&=0.\end{aligned}\right\}$$

We may, if we please, solve these as they stand and then plot the required points by the values of the vector $z=x+iy$, or we may use their z, ω equations, and eliminate ω between them. The resulting equation in z will, in either case, give the required points, which will include the foci and any double points the curve may have.

N.B.—Since $\frac{du}{dx}+i\frac{du}{dy}=0$ is satisfied by the coordinates of the imaginary points on the curve corresponding to the foci, it follows that $\frac{du}{dx}-i\frac{du}{dy}=0$ is satisfied by the coordinates of the conjugate imaginary points. Hence, both the foci and their images are given by

$$\left.\begin{aligned} u&=0,\\ \left(\frac{du}{dx}\right)^2+\left(\frac{du}{dy}\right)^2&=0,\\ z&=x+iy,\end{aligned}\right\}$$

and any curve of the form

$$\left(\frac{du}{dx}\right)^2+\left(\frac{du}{dy}\right)^2=A_1u+A_2u^2+\dots$$

will have the same points as images.

We will apply this method to find the director circle and directrices, and also the foci and focal images of the conic

$$2x^2-2xy+2y^2-2x-8y+11=0\,; \quad\dots\dots\dots\dots(1)$$

$\left(\frac{du}{dx}\right)^2+\left(\frac{du}{dy}\right)^2=0$ gives

$$(2x-y-1)^2+(x-2y+4)^2=0. \quad\dots\dots\dots\dots(2)$$

If we eliminate the xy term between (1) and (2) we obtain for the director circle

$$x^2+y^2-4x-6y+9=0. \quad\dots\dots\dots\dots(3)$$

If from (1) and (2) or from (1) and (3) we make the second degree terms a perfect square, which can be done in two ways, we obtain the directrices $(x+y-8)(x+y-2)=0$, and a pair of imaginary directrices (parallel to the major axis) $x-y+1=\pm i$.

Solving these simultaneously, we obtain four points whose vectors are

$$z=i,\quad z=1+2i,\quad z=3+4i,\quad z=4+5i.$$

Hence the two inner points, viz. (1, 2) and (3, 4), are the foci; and their images are (0, 1) and (4, 5).

The equation of the major axis is $x-y+1=0$, and that of the minor axis is $x+y-5=0$, these being the lines midway between the two imaginary and the two real directrices respectively.

Applying the same method to the general conic

$$ax^2+2hxy+by^2\mp 2gx+2fy+c=0$$

we obtain for the director circle

$$\left(\frac{du}{dx}\right)^2+\left(\frac{du}{dy}\right)^2=4(a+b)u,$$

which reduces to the normal form

$$C(x^2+y^2)-2Gx-2Fy+A+B=0.$$

The real and imaginary directrices are given by

$$\left(\frac{du}{dx}\right)^2+\left(\frac{du}{dy}\right)^2=4\lambda u,$$

where λ has one or other of the values given by the quadratic equation

$$(a-\lambda)(b-\lambda)=h^2.$$

If we wish to obtain the foci *without* their images, we eliminate x, y between

$$u=0,$$

$$\frac{du}{dx}+i\frac{du}{dy}=0,$$

$$x+iy=z,$$

and find for the vector equation of the foci

$$Cz^2-2(G+iF)z+(A+2iH-B)=0.$$

[NOTE.—In the above, the capital letters $A, B, \ldots$ denote the first minors of $a, b, \ldots$ in the discriminant of the conic.]

In the foregoing work we have tacitly assumed that the position of the point P which represents an imaginary point I of any curve is fixed relatively to the curve, *i.e.* is independent of any particular system of rectangular axes. It is perhaps desirable to prove this formally. This we shall do by showing that the position of the representative point is not affected by (1) changing the origin, (2) rotating the axes.

(1) *Move the origin to* (h, k).

Let (a, β) be the original coordinates of P, and (x, y) the original coordinates of the corresponding imaginary point I, so that $x+iy=a+i\beta$.

Then the new coordinates are respectively $(a-h, \beta-k)$ and $(x-h, y-k)$. We have to show that these continue to possess equal vectors.

Now, the new vector of I is $(x-h)+i(y-k)$ which

$$=(a-h)+i(\beta-k) \text{ since } x+iy=a+i\beta.$$

Therefore the new vector of P is equal to the new vector of I.

Hence P is still the representative of I.

(2) *Rotate the axes through the angle* ϕ.

Let $(r\cos\theta, r\sin\theta)$ be the original coordinates of P. Then its new coordinates will be

$$\{r\cos(\theta-\phi), r\sin(\theta-\phi)\}.$$

Now if (x, y) are the original coordinates of I, and (X, Y) its new coordinates, we have

$$X=x\cos\phi+y\sin\phi,$$
$$Y=y\cos\phi-x\sin\phi;$$

also
$$x+iy=r\cos\theta+ir\sin\theta.$$

Now
$$\begin{aligned}X+iY&=x\cos\phi+y\sin\phi+i(y\cos\phi-x\sin\phi)\\&=(x+iy)(\cos\phi-i\sin\phi)\\&=r(\cos\theta+i\sin\theta)(\cos\phi-i\sin\phi)\\&=r\cos(\theta-\phi)+ir\sin(\theta-\phi).\end{aligned}$$

Hence, in this case also, the point P is still the representative of I.

Q.E.D.

A. LODGE.

DRAFT SUGGESTIONS OF THE SUB-COMMITTEE ON THE TEACHING OF MECHANICS.

A. *Preliminary Experimental Work.*

(i) As it is specially important that in this subject the ideas should correspond with the facts, simple experiments would be useful, designed to illustrate

(α) composition and resolution of forces,

(β) the turning effect or moment of a force—in other words the principle of the lever,

(γ) friction (especially on an inclined plane).

(ii) The experiments should, if possible, aim at discovery and be quantitative—mere verification is less useful.

B. *General Remarks on Examples and Methods.*

(i) Examples should at first be almost exclusively numerical, and should in any case conclude with a numerical application.

(ii) Examples should as far as possible be practical. Those examples which specify the bodies on which the forces act are preferable to those which do not do so.

(iii) A specially instructive class of example consists in compiling a table or drawing a graph to shew the effect on a certain result of a variation in a certain datum.

(iv) The difficulties of beginners are often caused by undue haste to write down equations. It is essential to first consider the questions: What is the body whose equilibrum is being considered? What are the forces which act upon it?

(v) Stress should be laid on the great importance of checking results by an independent method; in particular questions should often be worked out both graphically and by calculation.

(vi) Simplifying assumptions such as that friction, stiffness of ropes, weights of certain bodies, etc., may be disregarded cannot be too explicitly stated.

(vii) Fancy names for technical terms are to be avoided.

(viii) Great prominence should be given to geometrical methods, to cure the prevalent devotion to analytical methods, a devotion which has obscured a great deal of the simplicity of dynamics.

C. *Statics.*

(i) As the basis of the subject the parallelogram of forces should be assumed as an experimental result.

(ii) This should be immediately followed by problems on three forces, to be solved by graphic methods.

(iii) The calculation methods should follow immediately on the graphic methods, and should be applied to numerical cases in which four-figure tables should be used, the angles 30°, 45°, 60° playing a very small part.

(iv) The problem of Parallel Forces should be attacked as follows [alternatives to be considered]: (α) By taking moments, assuming that the turning effect of a force is measured by its moment. (β) By defining a couple and proving (from the parallelogram of forces) that it is measured

by its moment. (γ) By the use of the funicular polygon. (δ) By the geometrical device which deduces parallel from non-parallel forces.

(v) It should be pointed out that all composition of forces assumes the existence of a rigid body to which the forces are applied, and that, failing the existence of such a body, composition of forces is unlawful and indeed unmeaning.

(vi) Machines. (α) The phrase 'Mechanical Advantage' should be either made definite or not used. (β) The three systems of pulleys should be dropped and special cases dealt with independently. (γ) The 'work done' should be an essential part of the discussion of machines, and attention should at an early stage be given to 'velocity ratio' and 'efficiency.'

(vii) The graphic method of dealing with problems on the equilibrium of bodies acted on by more than three forces, not necessarily meeting in a point, should be given.

(viii) The impression that the weight of a body really does act at its centre of gravity should be guarded against. This and other cases where rigidity is assumed should be impressed on the beginner by contrasting bodies which are not rigid.

(ix) It should be clearly brought out, by examples, that all the results of statics apply to cases of uniform motion.

D. *Dynamics.*

(i) Velocity. The meaning of the phrase 'velocity at a point' should be carefully brought out, by means of the idea of 'average velocity.' Average velocity should be defined as $\frac{\text{total distance}}{\text{total time}}$ and be carefully distinguished from

(α) the average of velocities at equal intervals of time,
(β) " " " space,
(γ) " the initial and final velocities.

(ii) Angular velocity should receive more notice as in connection with it a great variety of interesting examples arise, *e.g.* on the gearing of wheels.

(iii) The idea of work done by a force, with examples of work done against gravity and friction, should precede acceleration.

(iv) Acceleration. The velocity at any time should be represented geometrically. This should be used to illustrate the idea of acceleration and the formula for uniformly accelerated motion should be obtained from the fact that the graph is in this case a straight line.

(v) The formulae of uniform acceleration, having been proved as above, should, when possible, be used in the form 'average velocity = velocity at middle instant,' and thus the writing down of the formulae and substituting numbers for letters avoided.

(vi) Much more stress should be laid on Newton's first law and many examples given.

(vii) Newton's second law should be stated in modern terms only.

(viii) The word 'mass' should be introduced at as late a stage as possible, all elementary problems on forces producing accelerations being solved by simple proportion :

$$\frac{\text{acceleration produced}}{g} = \frac{\text{force acting}}{\text{weight of body}},$$

from the fact that a body's own weight produces acceleration g.

(ix) The word 'poundal' should be dropped. We have at first the

'pound' weight unit of statics and practical dynamics. Later, when absolute units are used, the dyne will naturally be employed.

(x) When, and if 'mass' is introduced, it should not be defined by the meaningless phrase 'quantity of matter,' but as (a) ratio of two masses is as inverse ratio of accelerations produced in them by the same force, assuming that a force can be duplicated, or ratio of two masses = inverse ratio of accelerations produced in them by a stress between them.

(xi) In dealing with the parallelogram of velocities it should be expressly stated that we are combining the velocity of A relative to B with that of B relative to the earth (as in all practical cases).

(xii) The parallelogram of forces will cause no difficulty if it is based on that of velocities.

This should rather be regarded as an illustration of a result which has already been arrived at experimentally.

(xiii) It should be pointed out that all the parallelogram laws are cases of a single law: that of the addition of vectors.

(xiv) There should be no objection to expanding the idea of a 'rate' and so leading up to the elementary ideas of the calculus (differentiation of x^n, $\sin x$, $\cos x$).

(xv) Atwood's machine should be regarded as typical of a set of problems rather than as a method of finding g and detailed descriptions of the machine should be omitted. The unsound method [mass moved $= m + m'$, moving force $= (m - m')g$] is to be condemned, and may be exposed by replacing one of the suspended masses by a small pulley which itself supports two unequal masses by a cord.

(xvi) The phrase 'centrifugal force' should be dropped.

(xvii) Having regard to the importance in physics of simple harmonic motion, and of preventing the notion that acceleration is always uniform, it is advisable to introduce simple harmonic motion and the pendulum earlier, before the more difficult work on projectiles and oblique collisions.

(xviii) Such problems as arise on the motion of a flywheel should form part of a course on elementary mechanics.

(xix) A full exposition of the nature of 'impulse of a force' should be given so that the nature of a blow, as quite different from a force, should be learned very early.

(xx) The restricted nature of the applicability of the principle of work and energy, and the unrestricted nature of that of the principle of impulse and momentum, should be copiously illustrated.

E. *Order of Teaching.*

(i) A short course of easy numerical trigonometry should precede the study of mechanics.

(ii) While several suggestions as to the order of (a) Statics, (b) Dynamics have been made above, the sub-committee does not wish to recommend that Statics should precede Kinematics, or *vice versa.*

Added subsequently.

When the equilibrium of two or more connected bodies is considered, the Principle of Separate Equilibrium should be distinctly enunciated. This is the suppressed premiss in all dynamical reasoning.

When the time comes to consider how Newton's 2nd Law, or its equivalent, is to be reconciled with the relativity of motion, a student should have little or nothing to unlearn.

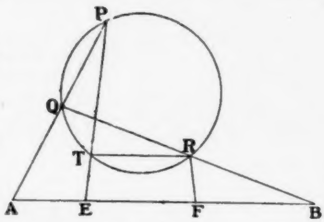
[It should be noted that some of the above are individual suggestions of members of the sub-committee.]

MATHEMATICAL NOTES.

139. [P. 3. b.] *Geometrical Note on Inversion.*

Let A, B be fixed points, P a variable point; $AQ.AP=k^2$ and $BR.BQ=k'^2$ so that R is the inverse of the inverse of P.

About PQR describe a circle and draw the chord RT parallel to AB. Produce PT to meet AB in E.



Then $P\hat{E}B=P\hat{T}R=P\hat{Q}R(P\hat{Q}B)$; therefore P, Q, E, B are concyclic and $AE.AB=AQ.AP=k^2$. Therefore E is a fixed point.

From R draw RF making $R\hat{F}B=T\hat{E}A$ so that $TEFR$ is a symmetrical trapezium and $FR=ET$.

Then $R\hat{F}B=T\hat{E}A=P\hat{T}R=P\hat{Q}R$ and $R\hat{F}A=A\hat{Q}R$; therefore Q, A, F, R are concyclic and $BF.BA=BR.BQ=k'^2$. Therefore F is a fixed point.

Again from the similar triangles PAE, BRF

$$EA:EP=FR:FB.$$

wherefore $\qquad FR.EP$ (or $ET.EP$) $=EA.FB=$ constant.

Hence if P describe any curve, R describes the geometrical image in the perpendicular bisector of EF of the inverse of the curve with respect to E.

Thus no matter what centres and constants of inversion are taken, repeated inversions of any plane curve will only give different inversions of the original curve. R. F. Davis.

140. [V. 1. a.] *On decimalisation of money.*

It is easy to calculate the exact amount of error in multiplication. Since 6d. is exactly £·025, it follows that the error in putting

$$\tfrac{1}{4}\text{d.}=£{\cdot}001 \text{ is } \tfrac{1}{24} \text{ of } £{\cdot}001=\tfrac{1}{24} \text{ of } {\cdot}001\times 20\times 12 \text{ pence}$$
$$={\cdot}01 \text{ pence.}$$

Again, in putting £3. 8s. $8\frac{3}{4}$d. = £3·436, there is no error in putting £3. 8s. 6d. = £3·425, but in putting $2\frac{3}{4}$d. = £·011 the error is

$$11\times{\cdot}01 \text{ pence}={\cdot}11 \text{ pence.}$$

Take the question: Express £3. 8s. $8\frac{3}{4}$d. as a decimal of a £1 to three figures and use your result to obtain 465 times £3. 8s. $8\frac{3}{4}$d.

We have £3. 8s. $8\frac{3}{4}$d. = £3·436 approx.

```
   £3·436
    465
 ────────
  1374·4
   206·16
    17·180
 ────────
£1597·740
```

= £1597. 14s. $9\frac{1}{2}$d. + an amount which is less than $\frac{1}{4}$d.

But the error is $11\times{\cdot}01\times 465$ pence
= 51·15 pence
= 4s. $3\frac{1}{4}$d. very nearly.

Hence the answer to the multiplication is really £1597. 19s. $0\frac{3}{4}$d. which agrees with what we obtain by ordinary multiplication. W. O. Hemming.

141. [**K. 17. b.**] *Some Simple Problems in Astronomy.*

Let ABC be a spherical triangle on the celestial sphere. First let B be the north pole, C the pole of the ecliptic, A a star whose longitude is less than 90° (a star whose longitude is greater than 90° may be similarly treated). Then BC=the obliquity of the ecliptic, $BA=90°$ − star's north declination, $CA=90°$ − star's north latitude, $ABC=90°$ + star's right ascension, $BCA=90°$ − star's longitude. Therefore, if we are given any two of the four quantities, star's right ascension, declination, longitude, latitude, we can find the other two by solving the triangle ABC.

Again, let B be the zenith, C the north pole, A a star whose hour angle is less than 180°. Then $BC=90°$ − north latitude of observer, $CA=90°$ − star's north declination, $BA=90°$ − star's altitude, ACB=star's hour angle, $ABC=$ 180° − star's azimuth. Therefore, if we are given any three of the five quantities, latitude of observer, star's hour angle, declination, azimuth, altitude, we can find the other two by solving the triangle ABC.

HAROLD HILTON.

PROBLEMS.

511. [**K. 8. b.**] A_1, A_2, A_3, A_4 are four concyclic points; O_1, O_2, O_3, O_4 the orthocentres of the triangles $A_2A_3A_4$, etc. Show that the quadrilaterals $A_1A_2A_3A_4$, $O_1O_2O_3O_4$ are equal in all respects and the relation between them is mutual. W. F. BEARD.

512. [**K. 12. b. β.**] In Malfatti's Problem, if x, y, z are the radii of the three circles, each of which touches the other two and two sides of the triangle ABC, prove that

$$\sqrt{yz}=\tfrac{1}{2}r(1+\tan A/4).$$

A. C. DIXON.

513. [**K. 2. b.**] Show that the centre of any one of the four circles touching the sides of a triangle ABC can be reached from the circumcentre S by three steps perpendicular to the sides of ABC and each equal to the circum-radius. Hence show that the centroid of the points of contact of the in-circle with the sides lies on the line joining its centre I to S, and extend the theorem to the ex-circles. E. M. LANGLEY.

514. [**X. 4. b. β.**] Devise a graphic method which would give the two smaller roots of the equation $x^3-ax+b=0$, when a is positive. A. LODGE.

515. [**L¹. 1. c.; 17. d.**] If a hexagon H be circumscribed about one conic and inscribed in another C, then the tangents to C at the angular points of H form another hexagon inscribed in a conic, and the two hexagons have the same Pascal line. J. S. TURNER.

516. [**K. 4.**] Two vertices and the intersection of the symmedians of a triangle are given; construct the triangle. X.

517. [**D. 6. a.; A. 3. g.**] Determine $f(x)$ a rational integral function of x of the seventh degree such that $f(x)+1$ is divisible by $(x-1)^4$ and $f(x)-1$ by $(x+1)^4$, and find the number of the real roots of the equation $f(x)=0$. (O.)

518. [**A. 3. b.**] If a is one of the roots of $x^3+px^2+qx+r=0$, express $\dfrac{a-a}{a-b}$ in the form Aa^2+Ba+C, where A, B, C are functions of p, q, r, a, b. (O.)

519. [**R. 2. b.**] A right circular cone is cut by a plane so that the section is an ellipse where centre is C; prove that the centre of gravity of the portion of the surface cut off between the vertex and the plane lies on a line through C parallel to the axis of the cone. (O.)

520. [R. 9. a.] The two legs of a pair of steps are of equal lengths and of weights W, W' ($W > W'$), and the steps will just stand on a rough ground when the legs contain an angle 2α; assuming that the coefficients of friction for the two feet are the same, prove that this coefficient is equal to $\frac{W+W'}{W+3W'}\tan\alpha$. (O.)

SOLUTIONS.

433. [K. 13. c.] *a, b, c, d, are the sides taken in order and e, f, the diagonals of a skew quadrilateral; shew that*

$$\cos\widehat{bc}\cos\widehat{ad}-\cos\widehat{ca}\cos\widehat{bd}+\cos\widehat{ab}\cos\widehat{cd}=(c^2a^2+b^2d^2-e^2f^2)/2abcd.$$

To what does this reduce in plano? [Cf. 387.] C. E. M'Vicker.

Solution by R. F. Davis.

Let $ABCD$ be the quadrilateral having

$$AB=a;\ BC=b;\ CD=c;\ DA=d;\ AC=e;\ BD=f.$$

Complete the parallelogram $ABCE$. Let AC, BE intersect in O. Denote BE and DE by x and y.

We have from $\triangle BDE$, $\quad f^2+y^2=2OD^2+\frac{x^2}{2}$,

" $\triangle ADC$, $\quad c^2+d^2=2OD^2+\frac{e^2}{2}$,

" $\triangle ABC$, $\quad a^2+b^2=\frac{x^2}{2}+\frac{e^2}{2}$.

Now $\quad \cos\widehat{bc}\cos\widehat{ad}-\cos\widehat{ca}\cos\widehat{bd}+\cos\widehat{ab}\cos\widehat{cd}$

$$=\frac{1}{4abcd}\{(b^2+c^2-f^2)(a^2+d^2-f^2)-(c^2+a^2-y^2)(b^2+d^2-y^2)$$
$$+(a^2+b^2-x^2)(c^2+d^2-e^2)\}$$

$$=\frac{1}{4abcd}\{(b^2+c^2-f^2)(a^2+d^2-f^2)-(e^2+f^2-b^2-d^2)(e^2+f^2-a^2-c^2)$$
$$+(c^2+d^2-e^2)(a^2+b^2-e^2)\},$$

which reduces to $\quad \frac{c^2a^2+b^2d^2-e^2f^2}{2abcd}$.

In plano this gives

$$c^2a^2+b^2d^2-e^2f^2=2abcd \text{ (cosine of sum of two opposite angles)},$$

a well known extension of Ptolemy's Theorem.

434. [K. 21. β.] *Divide a given straight line into n equal parts by the use of a pair of compasses only.* E. M. Radford.

Solution by Isaac H. Turrell.

This problem may be enunciated thus: In a system of collinear equidistant points if any two A, A_n be given, the series can be constructed.

If with any centre O, and radius $O_1O=r$ (any convenient radius subject to condition $2nr>AA_n$), the other extremity O_2 of the diameter OO_2 can be constructed by the compasses. Hence the series of collinear equidistant points $O, O_1, O_2 \ldots O_n$ can be constructed. With A, A_n as centres describe circles with radii$=nr$ intersecting in P_n, Q_n. Circles with centres A, P_n and radii r, $(n-1)r$ touch in P_1 on AP_n. Hence the series of collinear equi-

distant points A, $P_1 \dots P_n \dots$ can be constructed, and also the equal system A, $Q_1 \dots Q_n \dots$ on the opposite side of AA_n.

Circles with centres P_1, $P_2 \dots$ and radii r, $2r \dots$ meet the equal circles with centres Q_1, $Q_2 \dots$ in A_1, $A_2 \dots$.

Solution by PROPOSER.

It is clearly sufficient to take n even, for the case of n odd is covered by dividing the straight line into $2n$ equal parts.

Let AA_1 be the given straight line. With centre A_1 and radius AA_1 describe a circle. By marking off the radius three times round the circumference from A we obtain the point A_2, which is clearly on AA_1 produced, and such that $AA_2=2AA_1$. By continuing the process as in the figure we can obtain a point A_n on the line produced such that $AA_n=n \cdot AA_1$.

Now with centre $A_{\frac{n}{2}}$ and radius $A_{\frac{n}{2}}A_n$ describe a circle, and with centre A_n and radius AA_1 describe another circle. Let these intersect in B. With centres A and A_n, and radii AA_n and AB, describe two circles intersecting in C. Finally with centre C and radius CA_n describe a circle cutting AA_1 in E. Then AE will be $\frac{1}{n}$th part of AA_1.

For calling AA_1 the unit length we have $AA_n=n$; $AB=\sqrt{n^2-1}$; and if CN be perpendicular to AA_n, $AN^2-A_nN^2=AC^2-A_nC^2=n^2-(n^2-1)=1$,

i.e. $$(AN+NA_n)(AN-NA_n)=1.$$

But $AN+NA_n=n$, $\quad AN-NA_n=AN-NE=AE$;

$$\therefore\ n \cdot AE=1, \quad i.e.\ AE=\frac{1}{n}.$$

435. [L^1. 10. a.] *If PM be the perpendicular from the point P of a parabola on the tangent AM at the vertex A, find the locus of the foot of the perpendicular from M on AP.* V. RETALI.

Solution by J. BLAIKIE; A. W. POOLE.

Let the perpendicular from M meet AP in Q and the axis in B. Then by similar $\triangle$s PNA, BAM, $\quad AB=\dfrac{AM \cdot PN}{AN}=\dfrac{PN^2}{AN}=4AS$;

$\therefore$ B is a fixed point on the axis and the locus of Q is a circle on AB as diameter.

463. [L^1. 17. e.] *Shew that the equation to a circumconic of the triangle ABC can be written in the form*

$$\frac{a}{p\alpha}+\frac{b}{q\beta}+\frac{c}{r\gamma}=0,$$

where p, q, r are the lengths of the focal chords parallel to BC, CA, AB.

J. J. MILNE.

Solution by R. F. DAVIS.

Let $\Sigma L/\alpha=0$ be the equation to any given conic circumscribing the triangle of reference ABC. Then it is known that the equation to TA (the tangent at A to the conic) is $M\gamma+N\beta=0$, whence

$$\sin TAB : \sin TAC=-\gamma : \beta=N : M.$$

Now if P be a point on BA produced, and $PA'C'$ a secant parallel to AC, $q : r=PA' \cdot PC' : PA \cdot PB=PE' \cdot PC' : PB^2$, if BE' be drawn parallel to AA' to meet PC' in E'. In the limit as P approaches A this becomes

$$q : r = AE \,.\, AC : AB^2, \text{ where } BE \text{ is parallel to } AT, \text{ and meets } AC \text{ in } E$$
$$= b \sin ABE : c \sin AEB$$
$$= b \sin TAB : c \sin TAC$$
$$= bN : cM.$$

Thus $M : N = b/q : c/r$; and similarly for $N : L$ and $L : M$.

Solution by PROPOSER.

The product of the segments of a focal chord varies as the length of the chord.

Let $(\alpha', \beta', \gamma')$ be the coordinates of focus S, QSR the focal chord parallel to BC.

Let $SQ = \rho$. The coordinates of Q are $(\alpha', \beta' - \rho \sin C, \gamma' + \rho \sin B)$.

The equation to the circumconic of ABC is

$$l\beta\gamma + m\gamma\alpha + n\alpha\beta = 0. \qquad (1)$$

Since Q is on the conic,

$$l(\beta' - \rho \sin C)(\gamma' + \rho \sin B) + m\alpha'(\gamma' + \rho \sin B) + n\alpha'(\beta' - \rho \sin C) = 0;$$
$$\therefore \quad -\rho^2 . \, l \sin B \sin C + \rho\{l(\beta' \sin B - \gamma' \sin C) + \alpha'(m \sin B - n \sin C)\}$$
$$+ l\beta'\gamma' + m\gamma'\alpha' + n\alpha'\beta' = 0;$$
$$\therefore \quad \rho_1\rho_2 = \frac{l\beta'\gamma' + m\gamma'\alpha' + n\alpha'\beta'}{-l \sin B \sin C};$$
$$\therefore \quad l = \frac{l\beta'\gamma' + m\gamma'\alpha' + n\alpha'\beta'}{-\sin A \sin B \sin C} \times \frac{\sin A}{\rho_1\rho_2}; \qquad l \propto \frac{\sin A}{QR};$$

$\therefore$ substituting in (1) we have

$$\frac{a}{p\alpha} + \frac{b}{q\beta} + \frac{c}{r\gamma} = 0.$$

COR. If the conic is a parabola,

$$\frac{a}{\sqrt{p}} + \frac{b}{\sqrt{q}} + \frac{c}{\sqrt{r}} = 0.$$

If the conic is a rectangular hyperbola,

$$\frac{\sin 2A}{p} + \frac{\sin 2B}{q} + \frac{\sin 2C}{r} = 0.$$

REVIEWS.

Mathematical Crystallography and the Theory of Groups of Movements. By HAROLD HILTON, M.A.

This book may be heartily welcomed as a substantial addition to British crystallographical literature. Its object has been to collect for the use of English readers those results of the mathematical theory of crystallography which are not proved in the modern text-books in our language. The author makes no claim to originality, yet there is a considerable amount of original matter in the book, besides many new methods of presenting known mathematical facts. The proofs given in Chap. I., § 3, § 4, with respect to stereographic projection are new, as are also those of Chap. III., § 3, for the formulæ for the transformation of the indices of crystal faces from one set of axes to another, and of § 9, § 10, referring to those for the calculation of the axial ratios and facial indices from goniometrical measurements. These are decidedly important contributions to the mathematics of the subject. The student should

be warned, however, that the practical crystallographer calculates his results in most cases in a much simpler manner. Indeed, it has been the misfortune of crystallography that students are so frequently scared from the subject by the formidable formulæ given in most treatises, whereas the actual calculations are in general very much simpler than they appear, and can usually be worked out quite readily with the aid of the most elementary spherical trigonometry, provided the student is grounded beforehand in the law of anharmonic ratios and the other few fundamental laws of crystallography. New proofs are further given in Chap. IV., § 18, § 19, for the rules relating to axes of symmetry, and a considerable part of Chap. VII. (§ 3–§ 8, and the concluding notes) concerning the coordinates of equivalent points is likewise original. The proofs in Chap. IX., § 9, and Chap. XI., § 2 and § 3, relating to facial tension and capillarity constants, as well as the section of Chap. XIX. on orthorhombic holohedry, are also noticeable additions to the original literature of crystallography.

But the main feature of the book, and one for which the author has earned the gratitude of every British crystallographer, is the admirable summary which is given of the geometrical theory of crystal structure, due to the united labours of Bravais, Jordan, Sohncke, Federow, Schoenflies, and Barlow. The present is a particularly opportune moment, following on the British Association Report (Glasgow meeting, 1901) on "The Structure of Crystals," for the summarisation of the work in this direction, for it would appear to be now fairly complete. Moreover, this subject is one which has hitherto been all but untouched by English writers, with the exception of Kelvin, Barlow, and Sollas. The notation employed is chiefly that of Schoenflies, and the book contains an epitome of the great work of that author, *Krystallsysteme und Krystallstructur*, much more readable than the latter and rendered more intelligible by numerous excellent and original figures. The same may be said of the work of Federow, whose prolific memoirs in the *Zeitschrift für Krystallographie* are by no means easy to follow. The net result of the investigations of these independent workers, in jointly yet by different methods establishing the existence of 230 different types of homogeneous structure, is set forth with conspicuous clearness, and the important fact that these types fall naturally into the 32 classes of crystal symmetry which have been experimentally discovered to exist is graphically brought home.

The book is a solid record of facts as regards the possible packing in space of the structural units of crystals—the chemical molecules or their simple aggregates—and a presentment of the indisputable mathematical foundation of these facts. Speculation concerning possible developments as regards the internal structure of the molecules themselves is avoided, yet suggestions of such inevitable developments in the future are aroused by a perusal of the later chapters of the book. Indeed, the latter appeal to the chemist quite as much as to the mathematician and physicist. A striking instance of the kind of problem upon which this new direction of enquiry into the homogeneous partitioning of space is likely to throw much light, has recently arisen. The writer of this review has been working on the relation

between the radicle ammonium, NH_4, on the one hand, and the metals of the alkalies, potassium, rubidium, and cæsium on the other. For it has been shown as the result of a study of the four isomorphous (or more strictly, eutropic) normal sulphates, that two ammonium groups replace the two atoms of potassium in the sulphate K_2SO_4 with scarcely more change in the angles and physical properties of the crystals, or in the separations of the centres of contiguous molecules along the three rectangular directions, than is observed when the two potassium atoms are replaced by two atoms of the next family analogue—rubidium; and the change is much less than when two cæsium atoms are introduced instead of potassium. That is to say, 8 additional atoms are packed into the structure without disturbing the symmetry, thus proving the existence of large interspaces either between the molecular structural units or the atoms composing them, or possibly both. Indeed, it is now time that the enquiry advanced still further, into the internal structure of the molecule itself. It has been fairly conclusively shown that in the series of sulphates just referred to the simple chemical molecules are the structural units, so that this case is not further complicated, as happens in many other cases, by an aggregation of chemical molecules going to form the structural unit. Barlow has already attempted something in the way of tackling the question of internal molecular structure, and it may be hoped that as great success may attend such efforts as has crowned the work on the homogeneous partitioning of space. Problems such as that of the bestowal of the atoms of the ammonium group are constantly arising in the course of the investigations of the crystallographer, and it is by the combination of his practical work on the one hand, with such mathematical and geometrical labours as are summarised in Mr. Hilton's book on the other hand, that the highest results are to be expected, in extending our knowledge of the fundamental nature of organised solid matter. It is from this point of view that this book is so particularly welcome at the present moment.

A. E. H. TUTTON.

Calculating Scale: a substitute for the Slide Rule. By W. KNOWLES, B.A., B.Sc. (E. &. F. Spon, price 1s. net.)

Mr. Knowles gives a logarithmic scale and a plain scale side by side, each 100 inches long in 20 sections each 5 inches long. From these scales three figures of number and logarithm can be read definitely and a fourth figure by estimation.

The author, in an explanatory introduction, develops the principal properties of logarithms with the aid of the scale; and brings out clearly, in a concrete manner, the nature of characteristics, a matter which often greatly confuses beginners. As an aid to the comprehension of logarithms the scale may prove very useful; but opinions will probably differ as to whether for actual use in computation it possesses much advantage compared to a card of four figure logarithms. Some years ago Messrs. Hachette of Paris published a similar scale designed by M. Dumesnil from which four figures of number and logarithm could be read definitely and a fifth figure by estimation: affording a degree of accuracy somewhat beyond four figure logarithms.

The scales of M. Dumesnil and of Mr. Knowles may be used in conjunction with a pair of dividers for the mechanical performance of various calculations after the manner of the original 'Gunter'; but they do not possess the characteristic advantage of a slide rule, to wit, that of furnishing at a single setting the answer to a whole series of numerical cases of a problem, such as "Read off distances in yards corresponding to any series of given distances in metres."

C. S. J.

Projection Drawing. By O. GUETH, M.E. (E. & F. Spon, price 3s. net.)

This work consists of twelve plates, without explanations, illustrating projections of prisms and pyramids, sections and developments of certain solids, and intersections of prisms, cylinders, and spheres.

The diagrams are well chosen, and the mode of lettering and the auxiliary projections and developments indicated seem well calculated to aid a student in grasping the essentials of descriptive geometry—particularly if he actually cuts the developments out in cardboard, and folds them up so as to form the solid represented. C. S. J.

Traité de Géométrie. By C. GUICHARD. Part II. Compléments. Pp. vii. and 430. 1903. (Nony, Paris.)

The addition of fresh matter to the "programmes" of the secondary and higher schools in France has necessitated a fresh sequel to M. Guichard's book on the elements of geometry. The additions are threefold: in Geometry, the orthogonal projection of a circle and the theorems of Dandelin; in Mechanics, the theory of vectors; and in Descriptive Geometry, central projections and the generation of conics by means of homographic pencils. To this the author adds transversals, poles and polars, coaxal circles, tangent circles, inversion, spherical geometry, homology, elementary geometrical conics, systems of four lines belonging to the same quadric, etc. Although the theory of vectors finds its principal applications in Mechanics it rightly takes its place in a volume such as this, inasmuch as it forms a body of doctrine of a purely geometrical character. The author has also found that it is an admirable introduction to the study of systems of lines. He defines conics by their elementary properties, and by Dandelin's theorems a conic may be taken as the projection of a circle; hence are deduced properties of conics which enable him to prove that the projection of a circle wherever its centre of projection may be is a conic. Thus he directly proves that the conic is the locus of intersection of the corresponding rays of two homographic pencils. The volume closes with a section on plane polygons, polyhedra, and the measurement of areas.

An Elementary Treatise on Conic Sections and Algebraic Geometry, especially designed for the use of beginners. By G. H. PUCKLE. Pp. vii. and 379. 1903. (Macmillan.)

The first addition of this book was published in 1854, shortly after the appearance of Dr. Salmon's classical treatise. The fifth edition (stereotyped) was issued in 1884, and the book has been reprinted with revisions in 1887, 1892, 1896, and now again in 1903. Mr. Puckle's "Conics" is therefore well known to the majority of our readers, and there is but one point to which attention may be drawn. In the present edition, pp. 281-288, we find collected simple methods of reduction of the equation of the second degree, and of finding the foci, axes, and directrices. Mr. Puckle's memory is short when he claims that "the equation to the directrix (Art. 294) has not, as far as I know, appeared in any other work." This is true enough if a "work" is a text-book. But the equation in question will be found on p. 61 of the 68th volume of the *Educational Times Reprint*, and that, too, under the name of G. H. Puckle.

Geometrische Aufgaben und Lehrbuch der Geometrie: Planimetrie, Stereometrie, Ebene und Sphärische Trigonometrie. Vol. II. Trigonometrie. By M. SCHUSTER. Pp. vii. and 112. 1903. (Teubner.)

The chief interest of this admirable collection of questions to English teachers is the large number devoted to the application of Trigonometry to practical purposes—physics, navigation, planimetry, mathematical geography, and astronomy. The questions appear to be carefully graduated, and the majority of them are practical applications in the subjects we have mentioned. It is needless to dwell upon the educative effect on the mind of the student when he has to apply to realities what theory he has learned. His horizon is widened, his interest is sustained, and the faculty of concentration, on the development of which his intellectual future depends, is being steadily cultivated.

Advanced Algebra for Colleges and Schools. By W. J. MILNE. Pp. 608. 1903. (American Book Co.)

This is a complete course beginning with easy problems soluble by the simplest of simple equations, and concluding with chapters on graphs of functions and the elementary theory of equations. It covers the same ground as the larger Hall and Knight or Smith, but the general impression given to the reader is that the chapters which form the "advanced" part of the book are rather sketchy, though as far as they go they are clear. Continued Fractions, Probabilities, and Permutations, etc., for example, occupy but twelve pages each, mainly of widely spaced type, and Theory of Numbers only nine. The space devoted to what follows the Binomial Theorem fills less than a third of the book. Graphs are not introduced at all in the elementary part of the work. The book is beautifully printed and got up, and the general treatment within the narrow limits, which are perhaps determined by the circumstances of the secondary schools of America, is clear and satisfactory.

Vorlesungen über Algebra. By Dr. GUSTAV BAUER. Pp. iv. and 375, 1903. (Teubner.)

This volume is published as a tribute to Professor Bauer on the occasion of his attaining his 80th birthday, by his friends of the Mathematical Society of Munich, etc. It consists of four chapters devoted to the general properties of equations and their solution. The last chapter deals with the theory of Determinants and their application to quadratic and bilinear forms. About twelve pages are given to the Galois theory.

Mathematischer Bücherschatz. Compiled by E. WÖLFFING. Vol. I. Pure Mathematics. Pp. xxxvi. and 416. 1903. (Teubner.)

This bibliographical dictionary, of all the mathematical text-books and monographs of importance in pure mathematics issued during the nineteenth century, will be found a valuable addition to the mathematical library. A useful critical introduction deals with previous bibliographies, catalogues, synopses, etc. The sections divide the whole subject in such detail that the list of books on any portion of the subject may be found in a few moments. With the name of each book is given the name of the publisher, the date of appearance, and the price. The English prices are sometimes given in marks and at other times in shillings, not that it matters much. We have tested the book pretty carefully and find it exhaustive and accurate—astonishingly so—considering the various languages, and the enormous number of books recorded. From the list of errata, the greatest difficulty seems to have been the initials of authors. We can add one to the list—R. H. for K. H. Graham. In the title of Russell's *Foundations of Geometry*, p. 186, for *Russel* read *Russell* and for *essai* read *essay*. Wolstenholme's collection of problems is omitted, though Laisant's is included. Under Hearn, G. W., p. 286, for the *2 corder* read *2nd order*. Under Heath, same page, for *Appollonius* read *Apollonius*. The list of authors fills over thirty pages of three columns. Dr. Wölffing is to be congratulated on having so successfully coped with this mass of material, and we look forward with pleasure to the second volume on Applied Mathematics, which may appeal to an even larger sections of the community than the first. We should add that where a work—Salmon's *Conics*, for example—has been translated into other languages, full particulars are given.

De l'Expérience en Géométrie. By C. DE FREYCINET. Pp. 178. 4 frs. 1903. (Gauthier-Villars.)

M. de Freycinet can find no *a priori* reasons for our geometrical concepts. The very name, Geometry, exhibits in a marked degree the manner in which those conceptions were first manifested. Buildings could not be erected, ground could not be measured, until the existence of rudimentary ideas about lines, angles, and areas. These ideas eventually found their finest expression in architecture. Indeed, it has been said that whatever genius a race has for pure geometry is invariably betrayed in the style of architecture it affects. Our concepts of the straight line, of space, of volume, tangency, etc., are all derived

from actual experience, or from an intuition proceeding, perhaps, from a long series of predispositions accumulated in the race. The empiric geometry of the Egyptians would seem to have been derived by intuition from looking at a figure. M. de Freycinet asks the question : Is geometry purely rational or is it partly experimental? Does it belong to Pure Mathematics or to Mathematical Physics? His answer is that it belongs to the former if the axioms are self-evident, and to the latter if they are based on experience. He treats his subject in connection with geometrical concepts, the axioms, and those propositions which deductive geometry sets itself to prove. He finds them all to be the outcome of experience and observation. His monograph is well worth reading for its charm of style, and should be found interesting to the mathematician and the philosopher alike. In connection with the discussion, it may not be amiss to quote the following from Mr. Russell's *Principles of Mathematics*. "The common desire for self-evident axioms is entirely mistaken. This desire is due to the belief that the Geometry of our actual space is an *a priori* science, based on intuition. If this were the case it would be properly deducible from self-evident axioms, as Kant believed. But if we place it along with other sciences concerning what exists, as an empirical study based on observation, we see that all that can be legitimately demanded is that observed facts should follow from our premisses, and, if possible, from no set of premisses not equivalent to those that we assume. No one objects to the law of gravitation as not self-evident, and similarly, when Geometry is taken as empirical, no one can legitimately object to the axiom of parallels except, of course, on the ground that, like the law of gravitation, it need be only approximately true in order to yield observed facts. It cannot be maintained that no premisses except those of Euclidean Geometry will yield observed results; but others which are permissible must closely approximate to the Euclidean premisses."

Kinematics of Machines. By R. J. DURLEY. Pp. viii. and 379. 17s. net. 1903. (Chapman & Hall.)

This is an excellent text-book, written under the influence of *Die praktischen Beziehungen der Kinematik zu Geometrie und Mechanik* and its predecessor. Reuleaux conceived of a mechanism as a chain made up of links, any one of which may be considered fixed. With this conception as basis, and taking account of the relative motion of the links as determined by the pairing of their elements, we are enabled to develop the whole kinematic theory of mechanisms. The book is too technical to be reviewed in these columns, but the teacher of mechanics might do worse than read the first half of the volume. From the first six chapters he will be able to construct a large number of examples to be worked out by his classes. Chapter III., for instance, on plane mechanisms containing only turning pairs, deals with quadric crank chains, virtual centres and centrodes, the skew pantagraph, Peaucellier cells, etc. In most cases the relations between the linear and angular velocities are obtained graphically and analytically.

The School Arithmetic. By W. P. WORKMAN. Pp. viii. and 495. 3s. 6d. 1903. (Clive.)

This is a school course adapted from the *Tutorial Arithmetic*. It is amplified by a large selection of miscellaneous examples arranged in carefully graduated papers, new examples in approximate methods, and an additional collection of miscellaneous problems. "Harder Problems" of the *Tutorial Arithmetic* have disappeared. It is undoubtedly the best arithmetic for schools on the market.

Spherical Trigonometry. By D. A. MURRAY. Pp. ix. and 114. 2s. 6d. 1902. (Longmans, Green.)

The matter of this little book is confined to what is requisite for attacking the solution of spherical triangles and the simple practical problems depending thereon. After a revision of the elements of spherical geometry, the attention of the student is at once directed to the right-angled spherical triangle. The section dealing with it and the explanations in the sections (pp. 84-93) dealing with the application of spherical trigonometry to astronomy are full and singularly clear.

Vectors and Rotors, with Applications. By O. HENRICI and G. C. TURNER. Pp. xv. and 204. 4s. 6d. 1903. (Arnold.)

Mr. Turner, who had for many years been assistant to Professor Henrici at the City and Guilds' Central Technical College, must have felt it a labour of love to throw into systematic form these charming lectures, which have been kept from the public far too long. For on all sides there are signs that vectors are becoming of more and more importance in the teaching of mathematics. We notice that the author has not explicitly abandoned his intention of writing a book on vector analysis. We badly want a simple introduction to the subject, which, with all due rigor of treatment, will be complete enough to supply the needs of applied science. As to the relation between the vector as defined and used in this volume and the vector of quaternions, we must refer the reader to Professor Minchin's review in *Nature*, October 29th, the critical remarks in which will be found most helpful to the teacher whose knowledge of the Hamiltonian system is limited.

Vectors are discussed in their application to geometry and to statics. In the worked-out examples and in the exercises we find most of the familiar properties in pure and co-ordinate geometry. For instance, there are sections dealing with: the orthocentre of a triangle, the complete quadrilateral, mechanisms for drawing a straight line, the Peaucellier cell, the Hart contra-parallelogram, etc. Then in statics we find nearly twenty pages devoted to the finding of mass-centres. Just as by means of the link-polygon we can find the mass-centre of any number of points or lines, so by the same means we can determine the mass-centre of any irregular area. But in addition to the latter method the author gives the method of finding the mass-centre of an area, when the area has been found by means of the planimeter. Chapter V. is devoted to a very clear discussion of stresses in frames and their vectorial treatment. It should prove a welcome change from the methods which have so long reigned unchallenged in our class-rooms. This chapter concludes with an explanation of the term reciprocal figures, and their relation to a frame and its stress diagram. Finally there is a short chapter on the application of vector formulae to trigonometry. But what is a Rotor? This term has been devised by the author to express a localised vector, viz., a vector restricted to lie in a fixed straight line. The vector may be moved anywhere parallel to itself, but the rotor can be only moved to and fro in a fixed straight line, and that particular line is called the axis of the rotor. The geometrical theory of rotors is then shown to apply to forces.

There is not much doubt as to the reception this clear presentation of the subject will receive at the hands of teachers. We hope that the members of the Committee of the Association who are now sitting in judgment on our methods of teaching Mechanics will not loose sight of the advantages of the treatment by vectors and rotors so lucidly and elegantly exhibited in this little book.

Elementary Graphs. By R. B. MORGAN. Pp. viii. and 76. 1s. 6d. 1903. (Blackie.)

This book is a satisfactory introduction to the more prominent features of graphs. The pages devoted to the graphs of statistics are excellent, and the author suggests that students should be encouraged to draw graphs of the statistics in which they are personally interested—scores at cricket, marks in class, laboratory results, rise and fall of the barometer and thermometer, etc. The diagrams are especially good.

Hermann von Helmholtz. By LEO KOENIGSBERGER. Vol. I., pp. xi. and 375; 8 m. Vol. II., pp. xiv. and 383; 10 m. Vol. III., pp. xi. and 142; 3 m. 1903. (Vieweg, Brunswick.)

This definitive biography of a great natural philosopher is a worthy tribute to a splendid memory. Here we find material for the formation of an opinion as to the part played in moulding so fine a nature by hereditary influences and environment. His first introduction to the laws of phenomena were obtained from the study of geometry. The properties of wooden blocks were the stimulus which gave him the ideas that astonished his teachers when he entered on formal geometry. But from the earliest date he was more attracted by physics than by either algebra or geometry. At school he was not what the average teacher would call a "good boy." Many a time, he tells us, when he ought to have been translating Cicero he was calculating under his desk the path of the rays in a

telescope. In fact, it was by means of theorems discovered even in those early years that he was eventually led to the discovery of the ophthalmoscope. Financial circumstances compelled him to abandon the wish of his heart, which was to devote himself to the study of physics. He was thrust into the medical profession, and in the Army Medical School he came under the influence of Müller. His first work on research was on the relation between the work done and the heat produced in muscular contraction. Three years later he was appointed Professor of Physiology at Koenigsberg, receiving the princely salary of £120 a year. There are many personal touches in his autobiography which reveal to us how his ideas came to him. Occasionally they came as an inspiration; sometimes silently and at the time unrecognised as of any value. Often they came with the hour of waking, reminding him of the entry in the diary of Gauss: "The law of induction discovered Jan. 23, 1835, at 7 a.m., before rising." In Heidelberg they came when he was climbing the wooded hill-sides under the genial influence of the warm sun.

The main body of the book is limited to the physiological and physical researches of Helmholtz. His first mathematical paper was on the integrals of hydrodynamical equations which correspond to vortex movements, and the next on the motion of the air in pipes with wide open ends. Two years later he wrote a paper on the friction of viscous fluids. He then began to experiment on the theory of sound, and investigated the mathematical theory of violin strings and organ pipes. His researches in physiology led him to study electricity, and this brought him to the study of electric oscillations, and the differential equations which are connected with the motion of electricity. After he was appointed to the Professorship of Physics at Berlin he restricted his activities to that subject, and his work was mainly confined to electro-dynamics. The last four years of his life were devoted to a continuation of his investigations in the mathematical theory of electricity. The book contains many admirable portraits.

The biography could hardly have been trusted to more competent hands. For the author had been honoured for many years with the friendship of this great man, who seemed to be at home in almost every department of human intellectual activity—philosophy, literature, art, and nearly every branch of science. We need only add that the family of the illustrious savant placed at the disposal of the author all the letters and papers in their possession which would serve to throw any light on the man and his work.

Essai sur la sommation de quelques séries trigonométriques. By E. Estanave. Pp. 112. 6 fr. 1903. (Hermann, Paris.)

This is a study, not from the purely theoretical point of view, of certain series which are useful in mechanics and physics. The author takes some 80 types, and shows how they can be summed. The special feature of this interesting little monograph is the way in which he displays to his readers the virtue of intuition. He guesses the formation of the coefficients, and then shows that the law holds good in the general case. It is a curious study of the play of the mathematical imagination.

Exercises in Arithmetic, Oral and Written. By C. M. Taylor. Part I., pp. 124, 16. Part II., pp. vi. and 118, 16. 1s. 6d. each. 1903. (Arnold.)

Part I. consists of a large number of examples in the four rules, each process being followed by a series of concrete examples.

In Part II. is a collection of short and easy examples for the beginner. Easy decimals are placed before vulgar fractions. Stress is laid on the expression of denominators in prime factors. Both parts would seem to be excellently adapted for their purpose, and we should add that the sets of examples are very carefully graduated.

Five-Figure Logarithmic and other Tables. By Alex. M'Aulay. Pp. ix. and 161. 2s. 6d. 1903. (Macmillan.)

This is a handy and admirably printed pocket volume. For many purposes tables of four figures are not quite sufficient, while tables of more than five figures are rarely required. There were already two sets of tables of five-figure logarithms: Galbraith and Haughton's and De Morgan's, the latter published for the Society for the Diffusion of Useful Knowledge. These, however, differ from

the author's volume, in that they do not contain what he supplies, viz. an extended set of proportional parts in the angles, and useful subsidiary tables. The notes at the beginning of the book will be found useful to the inexpert.

The Junior Arithmetic. By R. H. Chope. Pp. viii. and 470. 1903. (Clive.)

Mr. Chope, who, as a collaborateur with Mr. Workman in the preparation of the *Tutorial Arithmetic* reviewed in these columns (p. 207) has adapted portions of that work for the purposes of junior form. The order of the chapters is still retained, as is the method of treatment, but the parts more suitable for seniors are omitted. There is a considerable number of additional examples, which are likely to delay the pupil far beyond the time when he would with advantage be tackling the harder parts of the subject.

A Complete Short Course of Arithmetic: mainly practical. By A. E. Layng. Pp. viii. and 220. 1s. 6d. 1903. (Blackie.)

This is a good little book and may be cordially recommended as the work of an experienced teacher. The best chapters are those on vulgar and decimal fractions. The foot-notes bring an occasional smile to the lips, although it is quite right and proper that the attention of children should be drawn to the unreal nature of the way in which they are led to approach their subject. For instance; "This room is supposed to have no windows or doors." "The carpet is supposed to be patternless or there would be waste in matching the pattern."

Setsquares, Protractors and Scales, designed by Professor Low. (Longmans Green.)

These may be recommended as elegent and accurate. All are covered with celluloid, and are made of white wood, unusually tough. They are edged with ebonised wood or with transparent celluloid. They appear to be able to stand even the wear that boys will give them.

Experimental Science. Elementary Practical and Experimental Physics. By G. M. Hopkins. 23rd Edition. Two Vols. in one. Pp. xi. and 531, v. and 538. £1 1s. 1902. (Spon.)

There must be something good about a book that has reached a twenty-third edition, and it is hardly necessary to add that such a book must have largely increased in size since its first appearance. It is brought up to date, and may be commended as an excellent introduction to the elements of experimental science. As mathematics are, as far as possible, deliberately excluded the volume calls for no further consideration in these columns.

The Arithmetic of Elementary Physics and Chemistry. By H. M. Timpany. Pp. 74. 1s. 1903. (Blackie.)

This is a collection of easy questions on specific gravities, centres of gravity, temperatures of mixtures, and chemical equations. Many of the typical questions are worked out.

Notes on Analytical Geometry. An Appendix. By A. C. Jones. Pp. 171. 6s. net. 1903. (Clarendon Press.)

This "appendix" to the ordinary works on Analytical Geometry, by the Senior Mathematical Master at the Bradford Grammar School will be found to be very useful to those who are preparing boys for university scholarships. There are several features about it which single it out from anything of the kind which has yet appeared. Much attention is paid to the use of the single variable, and the application of the elementary theory of equations to questions in analytical geometry is developed in a most successful manner. The author impresses on the student the value of the equation of a straight line passing through a given point and having a given direction, and reduces most of the well-known equations to this form. The chapter on cubic curves will be heartily welcomed, especially as an adequate treatment of the simpler properties of the unicursal cubic, and also as an instance of the value of the method of attack by means of the single parimeter. The book should be of great value to a large class of students. It concludes with over two hundred examples, with answers and hints to solution.

CORRESPONDENCE.

THE LONDON MATHEMATICAL SOCIETY.

To the Editor of the "Mathematical Gazette."

DEAR SIR,—Attention has lately been called to the inconvenient change which has been made in the form of the *Proceedings* of this Society. On reference it will be found that the only intimation which the members in general received of any proposed change is contained in an announcement, in small print at the very end of the volume, in a part not circulated till the end of August. Most of the members would hardly have read that announcement before a very American-looking journal reached them, whose appearance and style compares very unfavourably with the familiar series of neat volumes edited by Mr. Tucker. It is true that the announcement in question was made orally at a meeting on June 11, but only sixteen members were present. It is not a little remarkable that the action of the Council has been allowed to pass without any protest having yet been made by the other members, who ought to have had some opportunity of expressing their views on a point of such interest and importance. It is not too late to reprint the parts already issued. In several other respects the London Mathematical Society is not taking the place it ought to take among English scientific societies. Its library is stowed away in a dark attic and is practically inaccessible. Its members are not styled "Fellows." It has no lack of contributors to its *Proceedings*, but it makes no effort to improve the status of mathematicians in this country in the way that is undoubtedly done for other branches of science by the leading societies in Burlington House.—Yours faithfully,
G. H. BRYAN.

NOTICE.

Professor Hudson's Saturday morning Lectures, to Teachers on the teaching of Mathematics, are postponed till next term, beginning January 23, 1904, 10 a.m., at King's College.

COLUMN FOR "QUERIES," "SALE AND EXCHANGE," "WANTED," ETC.

(1) **For Sale.**

The Analyst. A Monthly Journal of Pure and Applied Mathematics. Jan. 1874 to Nov. 1882. Vols. I.-IX. Edited and Published by E. HENDRICKS, M.A., Des Moines, Iowa, U.S.A.

[With Vols. V.-IX. are bound the numbers of Vol. I. of *The Mathematical Visitor.* 1879-1881. Edited by ARTEMAS MARTIN, M.A. (Erie. Pa.)].

The Mathematical Monthly. Vols. I.-III. 1859-1861 (interrupted by the Civil War, and not resumed). Edited by J. D. RUNKLE, A.M.

Proceedings of the London Mathematical Society. First series, complete. Vols. 1-35. Bound in 27 vols. Half calf. £25.

(2) **Wanted.**

Vols. I.-IV. *Mathésis.* 1881-1884.

The Messenger of Mathematics. Vols. 2, 15-20, 24, 25.

Tortolini's Annali. Vol. I. (1850), or any one of the first eight parts of the Volume.

Carr's Synopsis of Results in Elementary Mathematics. Will give in exchange: Whewell's *History* (3 vols.) and *Philosophy of the Inductive Sciences* (2 vols.), and Boole's *Differential Equations* (1859).

(3) Dr. Muir, The Education Office, Cape Town, will give Vol. 109, *Crelle's Journal*, to any member of the Mathematical Association whose set is without it.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO.

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W. E. HARTLEY, B.A.

LONDON
GEORGE BELL & SONS, YORK ST., COVENT GARDEN
AND BOMBAY

Vol. II., No. 1. 25 JANUARY, 1901. One Shilling Net.

THE ANNUAL GENERAL MEETING of the Association will be held on Saturday, January 19, 1901, at 2 p.m., at King's College, Strand, W.C. The President, Sir ROBERT BALL, F.R.S., will read a paper on "Some Contributions to Geometry from recent Dynamical Work." Professor M. J. M. HILL, F.R.S., will introduce a discussion on the teaching of Proportion in Geometry.

The Mathematical Gazette is issued in January, March, May, July, October, and December.

Agent in France :—A. HERMANN, Librairie Scientifique,
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Charnock's *Graphic Statics*, etc.; W. J. Dobbs, M.A.

Forsyth's *Differential Equations*, Part II.; Prof. W. H. Lloyd-Tanner, F.R.S., D.Sc.

BACK NUMBERS OF THE MATHEMATICAL GAZETTE.

THE change from quarto form was made with No. 7. *No. 8 is out of print.* A few numbers may be obtained from *the Editor* at the following prices, post free:

No. 7, 2s.; Nos. 9–18, 1s. each.
The set of 7, 9–18, 10s. 6d.

These will soon be out of print and very scarce.

THE MATHEMATICAL ASSOCIATION.

(*An Association of Teachers and Students of Elementary Mathematics.*)

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Its fundamental aim as now constituted is to make itself a strong combination of all mathematical masters and mistresses, in primary as well as secondary schools, who are interested in promoting good methods of teaching. Such an Association would become a recognized authority in its own department, and would exert an important influence on methods of examination.

General Meetings of the Association will be held in London at least once a term, and in other places if desired. At these Meetings papers on elementary mathematics will be read, and any member will be at liberty to propose any motion, or introduce any topic of discussion, subject to the approval of the Council.

Intending members are requested to communicate with one of the Secretaries. The subscription to the Association is 7s. 6d. per annum, and is due on Jan. 1st. It includes the subscription to the *Gazette*.

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Vol. II., No. 26. MARCH, 1901. One Shilling Net.

The Mathematical Association.

THE NEXT GENERAL MEETINGS will be held at King's College, Strand, London, W.C., *at Eight p.m. on Thursday, May* 2, 1901, and *Thursday, October* 3, 1901.

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Stolz's *Theory of Rational Numbers*; Prof. G. B. Mathews, F.R.S.

Weber's *Die Partiellen Diff.-Gleich. der Math. Physik*; Prof. G. B. Mathews, F.R.S.

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Vol. II., No. 27. MAY, 1901. One Shilling Net.

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PROCEEDINGS OF THE

EDINBURGH MATHEMATICAL SOCIETY.

Vol. XVIII., pp. 26, 104,

Session 1899-1900 (7/6). Williams and Norgate.

CONTENTS :—Collignon, Ed., *Note sur un Problème de Géometrie;* Crawford, G. E., *Elementary proof that* $\Sigma(a)/n > (\Pi a)^{1/n}$*;* L. Crawford, *The evaluations of a Determinant in the theory of Statistics and Elliptic Geometry;* Dougall, J., *The determination of Green's Function by means of Cylindrical or Spherical Harmonics;* Professor Gibson, *Note on Inequality Theorems; Note on proofs by Projection; Proportion: a Substitute for Euc. V.;* Laisant, C. A., *La section de raison;* Muirhead, R. F., *Dissection of any two triangles into mutually similar pairs; Potential theorems;* Peddie, W., *Elementary proof of Potential Theorems regarding uniform Spherical Shells;* Philip, W. E., *Euler's Theorem on Homogenous Functions extended; Addition Theorems in Trigonometry;* Russell, A. D., *Dissection of any two triangles, special case.*

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W. J. GREENSTREET, M.A.

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LONDON
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Vol. II., No. 28. **JULY, 1901.** **One Shilling Net.**

The Mathematical Association.

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Vol. XVIII., pp. 26, 104,

Session 1899-1900 (7/6). Williams and Norgate.

CONTENTS:—Collignon, Ed., *Note sur un Problème de Géometrie;* Crawford, G. E., *Elementary proof that* $\Sigma(a)/n > (\Pi a)^{1/n}$; L. Crawford, *The evaluations of a Determinant in the theory of Statistics and Elliptic Geometry;* Dougall, J., *The determination of Green's Function by means of Cylindrical or Spherical Harmonics;* Professor Gibson, *Note on Inequality Theorems; Note on proofs by Projection; Proportion: a Substitute for Euc. V.;* Laisant, C. A., *La section de raison;* Muirhead, R. F., *Dissection of any two triangles into mutually similar pairs; Potential theorems;* Peddie, W., *Elementary proof of Potential Theorems regarding uniform Spherical Shells;* Philip, W. E., *Euler's Theorem on Homogeneous Functions extended; Addition Theorems in Trigonometry;* Russell, A. D., *Dissection of any two triangles, special case.*

Vol. XIX., pp. 4, 77,

Session 1900-1901 (7/6).

CONTENTS:—Allardice, R. E., *On four circles touching a common circle. On the nine-point conic. On a cubic curve connected with the triangle;* Carslaw, H. S., *Oblique Incidence of a Train of Plane Waves on a semi-infinite Plane;* Chrystal, G., *Elementary Theorems on Surds;* Davis, R. F., *Focal Relations of a Bicircular Quartic;* Gibson, G. A., *An extension of Abel's Theorem on the continuity of a Power Series;* Jack, J., *Change of Axes. Director Circle;* Mair, D., *On the* n^{th} *root of a prime;* Muirhead, R. F., *Inequalities;* Third, J. A., *Triangles triply in perspective;* Tweedie, C., *Area of Triangle in Cartesians. Addition Theorem in Trigonometry.*

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BOOKS, ETC., RECEIVED.

First Stage Building Construction. By B. CUNNINGHAM. Pp. viii. 240. Organised Science Series. 2s. 1901 (W. B. Clive).

**Plane Geometrical Drawing.* By R. C. FAWDRY. Pp. xi. 185. 6s. net. 1901 (Spon).

Algebra for Junior Students. By TELFORD VARLEY. Pp. vi. 152. 1s. 1901 (Allman).

**The Elements of Euclid.* Book XI. By R. LACHLAN. Pp. 51. 1s. 1901 (Arnold).

**A South African Arithmetic.* By H. HILL. Pp. x. 328. 1900. 3s. 6d. (Methuen).

Leçons d'Algèbre. By MME. A. SALOMON. Pp. 181. 2 fr. 1901 (Nony, Paris).

Problèms de Baccalauréat. By H. VUIBERT. Pp. 524. 3rd edition. 1901. 5 fr. (Nony, Paris.)

Traité de Mécanique. By E. CARVALLO. Pp. 156. 1901. 2nd edition. 2 fr. 50. (Nony, Paris.)

Euclid XI. By R. LACHLAN. Pp. 50. 1s. 1901 (Arnold).

**Linear Groups with an exposition of the Galois Field Theory.* By L. E. DICKSON. Pp. x. 312. 12 m. Geb. (Teubner, Leipzig.)

* Will be reviewed shortly.

PROCEEDINGS OF THE

EDINBURGH MATHEMATICAL SOCIETY.

Vol. XIX., pp. 4, 77,

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Vol. II., No. 31. JANUARY, 1902. One Shilling Net.

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Vol. II., No. 32. MARCH, 1902. 1s. 6d. Net.

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The Constitution of the Committee formed to consider improvements that might be effected in the Teaching of Elementary Mathematics is given on the third page of the cover.

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Reviews by Professor E. B. Elliott, F.R.S.; Professor G. B. Mathews, F.R.S.; Professor H. W. Lloyd-Tanner, M.A.; T. J. I'a. Bromwich, M.A.; W. J. Dobbs, M.A.; J. H. Grace, M.A.; E. M. Langley, M.A.; W. H. Macaulay, M.A., and others.

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BOOKS, ETC., RECEIVED—*Continued.*

The Foundations of Geometry. By D. HILBERT. Trans. by E. J. TOWNSEND. Pp. viii., 140. 4s. 6d. 1902. (Open Court, Chicago; Kegan Paul.)

Elementary Geometry. By W. M. BAKER and A. A. BOURNE. Pp. viii., 212. 2s. 6d. 1902. (Bell.)

Examples in Algebra. By C. O. TUCKEY. Pp. vi., 178. 3s. 1902. (Bell.)

Easy Mathematical Problems. By C. DAVISON. Pp. vi., 119. 1902. (Blackie.)

Essentials of Plane and Solid Geometry. By W. WELLS. Pp. viii., 391. $1.25. (Heath.)

Famous Geometrical Theorems and Problems. By W. RUPERT. Parts I-IV. 40 c. Pp. 107. 1901. (Heath's Mathematical Monographs.)

Factoring. By W. WELLS. Pp. 30. 10 c. 1902. (Heath's Mathematical Monographs.)

Graphs. By R. J. ALEY. Pp. 21. 10 c. 1902. (Heath's Mathematical Monographs.)

Mensuration. By R. W. K. EDWARDS. Pp. vi., 304., xviii. 3s. 6d. (Arnold.)

Higher Arithmetic and Mensuration. Pp. vi., 304. 2nd edition. 3s. 6d. 1900. (Blackie.)

Philosophical Essay on Probabilities. By PIERRE SIMON, MARQUIS DE LAPLACE. (Translated by F. W. TRUSCOTT and F. L. EMORY.) Pp. vi., 196. 2 $. 1902. (Wiley, New York; Chapman & Hall.)

Discussion on the Teaching of Mathematics. (Brit. Assoc., 1901.) Edited by Professor PERRY. Pp. x., 126. 2s. net. 1902. (Macmillan.)

On an Inversion of Ideas as to the Structure of the Universe (The Rede Lecture, 1902). By O. REYNOLDS. Pp. 44. 1s. 6d. net. (Cambridge University Press.)

Gedächtnisrede auf I. L. Fuchs. By HAMBURGER. (Teubner.)

A Graphic Method for solving certain Questions in Arithmetic or Algebra. By G. L. VOSE. 2nd edition. Pp. 62. 50 c. 1902. (D. Van Nostrand, New York.)

I tre problemi classici degli antichi (I. La Quadratura del Cerchio). By Prof. BELLINO CARRARA, S.J. Pp. 172. (Fusi, Pavia.) 1902.

Anleitung zur Auflösung eingekleideter algebraischer Aufgaben. By E. BARDEY. Edited by F. PIETZKER. Pp. viii., 159. 2 m. 60. 1902. (Teubner.)

Abhandlung zur Geschichte der Mathematischen Wissenschaften. By A. A. BJÖRNBO, H. SUTER, and K. BOPP. Pp. viii., 338. 16 m. 1902. (Teubner.)

Trigonometrie. By M. SCHUSTER. Pp. viii., 112. 2 m. 1902. (Teubner.)

ERRATA.

No. 30, p. 115, l. 14 up, for X read P.
l. 12 up, for $-$ read $=$.

Corrections for the final form of the Report of the M. A. Committee on Geometry, Arithmetic, and Algebra.

(7) GEOMETRY.—*read,* "In the absence of any other authorised logical order of theorems it is not proposed to interfere with that of Euclid:—in other words, ..."

(9) ARITHMETIC.—*for* "and that" *read* ":—".

(15) *read* "That in many cases a rough estimate might usefully *precede* the detailed work of an arithmetical computation.

Delete 20; *for* 15, 16, 17, 18, 19, *read* 16. 17, 18, 19, 20.

(21) *Delete* "the rule for", and for "and that" *read* ":—"

(23) *add* "and quadratic equations."

(32) *read* "That algebraic work should be checked by the consideration of simple special cases and the substitution of numbers."

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NOTICE.

A Meeting of the Mathematical Association will be held at King's College, Strand, London, W.C., *at* 3 *p.m. on Saturday, May* 23*rd*, 1903. The following Papers will be read :—

1. "The Slide Rule and its use in teaching Logarithms," by C. S. Jackson, M.A.
2. "A Geometrical Note," by R. F. Davis, M.A.

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A Critical Exposition of the Philosophy of Leibnitz. By B. RUSSELL. pp. xviii., 311. 1900. (Cam. Univ. Press.)

The Principles of Mathematics. Vol. I. By BERTRAND RUSSELL. pp. xxx., 534. 12s. 6d. net. 1903. (Cam. Univ. Press.)

Plane and Spherical Trigonometry. By G. A. WENTWORTH. pp. 207, 25. Second edition. 1903. 4s. (Ginn.)

Traité de Géométrie. Part II. *Compléments.* By C. GUICHARD. pp. vii., 430. 6 frcs. 1903. (Nony.)

Exercises in Arithmetic (oral and written). By C. M. TAYLOR. Part II. 1903. 1s. 6d. (Arnold.)

Mathematics in the Elementary School. By D. E. SMITH and F. M. M'MURRY. pp. 70. 30 cts. (Teachers' College Record, Mar. 1903. Columbia Univ. Press.)

Annals of Mathematics. Edited by ORMOND STONE and others. April, 1903. 2s. (Longmans.)

The Cardioid and Tricuspid. Archibald. *Note on a Partial Differential Equation of the First Order.* Roe. *On a Generalisation of the Set of Associated Minimum Surfaces.* Gale. *Twisted Quartic Curves.* White. *On the Characteristics of Differential Equations. I.* Hedrick.

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The Journal of the College of Science, Imperial University of Tōkyō, Japan. Vol. XIX. Art. 5.

Ueber die im Bereiche der rationalen complexen Zahlen Abel'schen Zahlkörper. By Prof. T. Takagi.

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American Journal of Mathematics. Edited by FRANK MORLEY. Vol. XXV. No. 3. 1½ $. July 1903. (Kegan Paul.)

Isothermal Conjugate Systems of Lines on Surfaces. L. P. Eisenhart. *Some Differential Equations connected with Hypersurfaces.* G. O. James. *Forms of Sextic Scrolls of Genus greater than One.* V. Snyder. *Geometry on the Cuspidal Cubic Cone.* F. C. Ferry.

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Proceedings of the Edinburgh Mathematical Society. Vol. XXI. 1902-1903.

Mathematical Correspondence (R. Simson, Matthew Stewart, James Stirling). Dr. Mackay. *Proofs of Fourier's Series.* Dr. Carslaw. *Generalised Forms of the Series of Bessel and Legendre.* Rev. F. H. Jackson. *On Uniqueness of Solution of the Linear Differential Equation of the Second Order.* Dr. Peddie. *On Two Intrinsically Connected Plane Curves.* R. F. Davis. *On Anti-reciprocal Points.* A. G. Burgess. *On Singular Points of Plane Curves.* T. B. Sprague. *Triangles in Multiple Perspective.* J. A. Third. *Mechanical Construction of Quartic Trisectrix.* H. Poole. *Identities and Inequalities of Symmetric Algebraic Functions of* n *Letters.* R. F. Muirhead. *Etc.*

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PAPERS.

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(3) Models of Regular and Semi-regular Solids, including the four polyèdres étoilés of Poinsot. Mr. E. M. LANGLEY.
(4) Advanced School Courses of Mathematics. Mr. C. A. RUMSEY.

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Prof. T. J. I'a BROMWICH; Prof. A. LODGE; Prof. G. B. MATHEWS, F.R.S.; J. H. GRACE, M.A.; R. W. H. T. HUDSON, M.A.; E. M. LANGLEY, M.A.; W. P. WORKMAN, M.A.; H. W. RICHMOND, M.A.; A. N. WHITEHEAD, M.A.; E. W. BARNES, M.A.; F. S. MACAULAY, D.Sc., and others.

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Les Installations et Les Programmes de l'Institut Montefiore. Université de Liège.

Etude de quelques surfaces algébriques engendrées par des courbes du sécond et du troisième ordre. By Prof. M. STUYVAERT. Pp. 73. 2 fr. 50. 1902. (Hoste, Gand.)

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Magic Squares Easily Constructed. By F. NEWCOME. (*Insurance Observer.* Oct. 23rd. 1903.)

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